

**PONDICHERRY UNIVERSITY**  
**RAMANUJAN SCHOOL OF MATHEMATICAL**  
**SCIENCES**

**DEPARTMENT OF MATHEMATICS**  
**M.Sc. MATHEMATICS PROGRAMME**

**SYLLABI**  
**WITH EFFECT FROM THE ACADEMIC YEAR**

**2023-24**

**(APPROVED)**

## **PREAMBLE AND OBJECTIVES:**

The Choice Based Credit System (CBCS) is being introduced for M. Sc Mathematics from the academic year 2023-2024 in accordance with the directives of University Grant Commission (UGC). The system provides an opportunity to students to design curriculum to self- individual needs.

CBCS in M.Sc Mathematics is aimed at

- Offering courses on credit mode and enrich the quality of teaching – learning at higher education level.
  
- Encouraging faculty to design and develop newer soft core courses.
  
- Enabling students to make a choice between different streams of soft core courses.

## M.Sc. MATHEMATICS

### List of Hard Core Courses (to be) offered from the Academic Year 2023-2024

SL. NO.	COURSE CODE	COURSE TITLE
1	MATH-411	Advanced Algebra
2	MATH-412	Real Analysis – I
3	MATH-414	Topology
4	MATH-424	Ordinary Differential Equations
5	MATH-421	Linear Algebra
6	MATH-422	Lebesgue Measure Theory
7	MATH-423	Complex Analysis
8	MATH-425	Real Analysis – II
9	MATH-513	Functional Analysis
10	MATH-516	Theory of Numbers
11	MATH519	Galois Theory
12	MATH- 537	Partial Differential Equations

## M.Sc. MATHEMATICS

### List of Soft Core Courses to be offered from the Academic Year 2023 -24

SL. NO.	COURSE CODE	COURSE TITLE
1.	MATH 413	Discrete Mathematics
2.	MATH 415	Continuum Mechanics
3.	MATH 416	Graph Theory
4.	MATH 417	Scilab Practical
5.	MATH-511	Fluid Mechanics
6.	MATH-512	Differential Geometry
7.	MATH-514	Analytical Dynamics
8.	MATH-515	Fuzzy Sets and its Applications
9.	MATH-522	Algebraic Graph Theory
10.	MATH-523	Algorithmic Graph Theory
11.	MATH-527	Algebraic Number Theory
12.	MATH-528	Advanced Algebraic Number Theory
13.	MATH 529	Theory of Fuzzy sets
14.	MATH-531	Cryptography
15.	MATH-533	Advanced Topics in Topology and Analysis
16.	MATH-534	Approximation Theory
17.	MATH-536	Difference Equations
18.	MATH-538	Lie Groups of Transformations and Differential Equations
19.	MATH-539	Numerical Analysis for Ordinary Differential Equations
20.	MATH-540	Advanced Fluid Mechanics
21.	MATH-541	Integral Equations
22.	MATH-542	Advanced Mathematical Analysis
23.	MATH-544	Elements of Harmonic Analysis
24.	MATH-545	Linear Lie Groups
25.	MATH-547	Advanced Functional Analysis
26.	MATH-549	Commutative Algebra
27.	MATH-551	Functional Analysis- II
28.	MATH-552	Operator Theory
29.	MATH-554	Non-Commutative Rings and Representations
30.	MATH-555	Advanced Complex Analysis
31.	MATH-559	Mathematical Practical
32.	MATH-560	Mathematical Software
33.	MATH-562	Numerical Analysis
34.	MATH-563	Integral Transforms
35.	MATH-564	Discrete Dynamical Systems
36.	MATH-565	Dynamical Systems
37.	MATH-566	Advanced Topology
38.	MATH- 567	Special Functions in Number Theory
39.	MATH- 568	Theory of Partitions
40.	MATH - 570	Introduction to Fuzzy Set
41.	MATH- 571	Calculus of Variations
42.	MATH- 572	Probability and Statistics

**M.Sc. MATHEMATICS**  
**HARD CORE : MATH 411- ADVANCED ALGEBRA (4 credits)**

	Course Outcome	Level
CO 1	have a thorough introduction to the subject	Understand
CO 2	appreciate Sylow's theorems and its applications	Apply
CO 3	solve problems on conjugacy classes, Sylow's theory, field extensions and solvable groups	Analyze
CO 4	Find the dimension of the constructed extension fields	Evaluate
CO 5	have a detailed knowledge on Group theory	Create

**Objectives:**

To study Group Actions, The Sylow theorems, Direct and semi-direct products, Ring Homomorphism and Polynomial rings.

**Unit I:** The isomorphism theorems -Composition Series - Transpositions and Alternating groups-

**Unit II:** Group Actions: Group Actions and Permutation representations-Group acting on themselves by left multiplication-Cayley's theorem

**Unit III:** Group acting on themselves by conjugation -The class equation- Automorphisms- The Sylow theorems- The simplicity of  $A_n$ .

**Unit IV:** Direct and semi-direct products and abelian groups: Direct products- The fundamental theorem of finitely generated abelian groups -  $p$ -groups, Nilpotent Groups, and Solvable Groups

**Unit V :** Polynomial Rings - Definitions and Basic Properties - Polynomial Rings over Fields I - Polynomial Rings that are Unique Factorization Domains - Irreducibility Criteria.

**Text Book:**

Abstract Algebra (Third Edition) by David S. Dummit and Richard M. Foote, Chapter 3 (Section 3.3 to 3.5, except Holder program), Chapter 4, Chapter 5 (Sections 5.1 and 5.2 only), Chapter 6 (Section 6.1), Chapter 10 (Section 10.1 to 10.3).

**Reference books**

- 1 M. Artin: Algebra, Prentice-Hall of India, 1991.
2. I.N.Herstein: Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
3. N.Jacobson: Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
4. S.Lang: Algebra, 3rd edition, Addison-Wesley, 1993.

**Mapping of Program Specific Outcomes with Course Outcomes**

CO / PSO	PSO1	PSO2	PSO3	PSO4	PSO5	PSO6	PSO7	PSO8
CO 1	1	1	0	1	0	0	0	0
CO 2	1	1	0	1	0	0	0	0
CO 3	1	1	0	1	0	0	0	0
CO 4	1	1	0	1	0	0	0	0
CO 5	1	1	0	1	0	0	0	0

**M. Sc. MATHEMATICS**  
**HARD CORE: MATH-412: REAL ANALYSIS – I (4 credits)**

	<b>Course Outcome</b>	<b>Level</b>
<b>CO 1</b>	explain the concepts of infimum, supremum and metric spaces	Remember & Understand
<b>CO 2</b>	demonstrate the convergence of series and power series using various tests	Apply
<b>CO 3</b>	analyze the topological properties of continuous functions	Analyze
<b>CO 4</b>	determine the interior point, limit point, closure of subsets of various metric spaces and also the limits of functions, sequences and subsequences	Evaluate
<b>CO 5</b>	construct functions that have various combination of the properties continuity, uniform continuity and differentiability	Create

**Objectives:**

To study basic definitions on countability, compactness and connectedness, convergence of sequence and series, limits, continuous functions, derivatives, Riemann- Stieltjes integral.

**Unit-I :** Finite, countable and uncountable sets - Metric spaces - Compact sets - Perfect sets - Connected sets - Convergent sequence - Subsequences - Cauchy sequences - Upper and lower limits – Some special sequences.

**Unit- II :** Series- Series of non- negative terms - The number  $e$  – The root and ratio tests - Power series - Summation by parts - Absolute convergence – Addition and multiplication of series - Rearrangements of series.

**Unit- III :** Limits of functions - Continuous functions - Continuity and compactness - Continuity and connectedness - Discontinuities - Monotonic functions - Infinite limits and limits at infinity.

**Unit- IV :** The derivative of a real function - Mean value theorems – The continuity of derivatives - L'Hospital's rule - Derivatives of higher order - Taylor's theorem - Derivatives of vector – valued functions.

**Unit- V :** The Riemann- Stieltjes integral- Definition and existence of the integral - Properties of the integral - Integration and differentiation - Integration of vector- Valued functions - Rectifiable curves - Improper Riemann Integrals.

**Text Book**

Walter Rudin, Principles of Mathematical Analysis- McGraw Hill International Editions, Mathematics series, 1976 (Chapters 2-6)

**Reference Books**

1. Patrick M. Fitzpatrick, Advanced Calculus, AMS, Pure and Applied Undergraduate Texts, Indian Edition, 2<sup>nd</sup> edition, 2009.
2. Tom Apostol, Mathematical Analysis, Narosa Publishing House, Indian edition, 1985.
3. N.L.Carothers, Real Analysis, Cambridge University Press, 2000.
4. Karl.R.Stormberg, An Introduction to Classical Real Analysis, AMS Chelsea Publishing, 2015.
5. Richard R Goldberg, Methods of Real Analysis, Oxford and IBH Publishing Co. 1970.

**M.Sc. MATHEMATICS**  
**HARD CORE: MATH-414: TOPOLOGY (4 Credits)**

	Course outcome	Level
<b>CO 1</b>	understand the concepts of topology, basis, sub basis, subspace topology, open set, closed set, interior, closure, continuous function, homeomorphism, and open map.	Remember Understand
<b>CO 2</b>	find the applications of topology.	Apply
<b>CO 3</b>	identify the differences among the various separation axioms	Analyze
<b>CO 4</b>	discuss the proofs Urysohn's lemma, Tietze's extension theorem, Urysohn's metrization theorem, Tychonoff's theorem	Evaluate
<b>CO 5</b>	construct examples and counter examples of various topological properties	Create

**Objectives:**

To study Topological spaces, Bases, Compactness, Regular space, Normal Space and Connected space.

**Unit-I :** Revision of sets - Functions - Product of sets – Relations – Countable sets – Uncountable sets – Partially ordered sets and lattices – Metric spaces – Definition and examples – Open sets and closed sets in metric spaces – Open subsets of real line.

**Unit -II :** Topological spaces -- Definitions and examples - Closure and related concepts – Open bases and open sub bases – Separability and second countability - Lindloff's Theorem

**Unit-III :** Compactness – Basic results -- Continuous maps on compact sets - Characterization of compactness by basic and sub basic open covers – Tychonoff's theorem - Generalized heine – Borel theorem.

**Unit – IV :** Compactness for metric spaces – Sequential compactness - Lebesgue covering lemma - Sequential compactness and compactness coincide on metric spaces -  $T_1$  spaces - Hausdorff spaces.

**Unit -V :** Completely regular spaces and normal spaces – Urysohn's lemma and Tietze extension theorem- –Connected spaces – Components of a space.

**Text Book**

G. F. Simmons, an Introduction to Topology and Modern Analysis, McGraw-Hill Kogakusha, Tokyo, 1963 (Chapter 1 – Revision of Sections 1—3, Section 4—8. Chapter 2 – Sections 9 - 12 Chapter 3 – Sections 16, 17 and 18. Chapter 4 – Sections 21 – 24. Chapter 5 – Sections 26 - 28 Chapter 6 – Sections 31 and 32)

**Reference Books**

1. J. R. Munkres, Topology, Pearson Education Inc., Second Edition, 2000.
2. Stephen Willard, General Topology, Dover Publication 2004.
3. J. Dugundgi, Topology, Allyn and Bacon, Boston, 1966.
4. Fred.H.Croom, Principles of Topology, Dover publications, 2016.

**M.Sc. MATHEMATICS**  
**HARD CORE: MATH-421 LINEAR ALGEBRA (4 Credits)**

	Course Outcome	Level
CO 1	understand the concepts of vector spaces, subspaces and linear transformations	Understand
CO 2	appreciate the geometry of vector spaces using parallelogram law, Pythagorean theorem and triangle in equality	Apply
CO 3	know the relation between matrices and linear transformations	Analyse
CO 4	know the concepts of diagonalization, Jordan form and rational canonical form	Evaluate
CO 5	know the difference between various kind of operators like self-adjoint operators, Normal operators etc.	Create

**Objectives:**

To study Linear Transformation, Nilpotent Transformation, Canonical forms, Trace, Transpose, Determinants, Quadratic forms.

**Unit-I** (Sections – 6.1, 6.2 and 6.3) : The Algebra of linear transformations-Characteristic roots-Similarity of linear transformations, Invariant subspaces and matrices.

**Unit-II** (Sections – 6.4 and 6.5) : Reduction to triangular forms- Nilpotent transformations - Index of nil potency and invariant of nilpotent transformation.

**Unit-III** (Sections – 6.6 and 4.5): Jordan blocks and Jordan forms- Modules - Cyclic modules - Fundamental theorem on modules over PID.

**Unit-IV** (Sections - 6.7, 6.8 and 6.9): Rational canonical form- Trace- Transpose and Determinants.

**Unit-V** (Sections – 6.10 and 6.11): Hermitian - Unitary and Normal transformations - Real quadratic forms.

**Text Book:** I.N.Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

**Reference Books**

1. M.Artin, Algebra, Prentice-Hall of India, 1991
2. N.Jacobson, Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
3. S.Lang, Algebra, 3rd edition, Addison-Wesley, 1993
4. P. B. Bhattacharya, S. K. Jain and S.R. Nagpaul, Basic Abstract Algebra ( 2nd Edition) Cambridge University Press, Indian edition, 1997
5. Kenneth Hoffmann and Ray Kunze, Linear Algebra, (Second edition), Pearson, 2015
6. S. Friedberg, A.Insel and L.Spence, Linear Algebra, (4<sup>th</sup> Edition) Pearson, 2015.



## M.Sc MATHEMATICS

### HARD CORE: MATH422 - LEBESGUE MEASURE THEORY (4 Credit)

#### Objectives:

To introduce Algebras and  $\sigma$  – algebras, Measurable sets, Measurable functions, Integrable functions, Functions of bounded variation, Lebesgue integrability and Characterization of absolutely continuous functions as indefinite integrals.

#### Unit-I (Sections: 3.1, 3.2, 3.3, 3.6 from [1] and 2.1, 2.2 from [2])

Motivation to Lebesgue Measure Theory – General extension Theory – Algebra of sets – Examples – Finitely/Countably additive set functions – Ulam’s Theorem – Continuity from below/above of measures – The Lebesgue outer measure  $m^*$  - Examples – Properties

#### Unit-II (Sections: 2.3, 2.4, 2.5, 2.6, 2.7 from [2])

Lebesgue measurable sets – Examples – The set of all Lebesgue measurable sets  $\mathcal{M}$  is an algebra -  $m^*$  is finitely additive over  $\mathcal{M}$  –  $\mathcal{M}$  is a sigma algebra –  $m^*$  is a measure on  $\mathcal{M}$  – Outer and inner approximation of Lebesgue measurable sets by open and closed sets respectively – Continuity of the Lebesgue measure – Example of a nonmeasurable set – The Cantor Lebesgue function

#### Unit-III (Sections: 3.1, 3.2, 3.3 from [2])

Lebesgue measurable functions – Examples – Pointwise limit of sequence of measurable functions – Simple functions – The simple approximation Lemma – The simple approximation Theorem – Egoroff’s Theorem – Lusin’s Theorem

#### Unit-IV (Sections: 4.2, 4.3, 4.4, 4.5, 5.3 from [2])

The Lebesgue integral of a simple function – The Lebesgue integral of a bounded measurable function over a set of finite measure – Properties – The Bounded Convergence Theorem - The Lebesgue integral of a nonnegative measurable function – Properties – Chebychev’s inequality – Fatou’s Lemma – Monotony Convergence Theorem – The general Lebesgue integral – The Lebesgue dominated Convergence Theorem – Characterization of Riemann integrable functions – Improper Riemann integrals and their Lebesgue integrals

#### Unit-V (Sections: 6.1, 6.2, 6.3(upto6.3.6) of [1])

Review of functions of bounded variation – Absolutely continuous functions – Lebesgue’s Theorem on differentiability of monotony functions – The Lebesgue singular function – Fundamental Theorem of Calculus [I and II] for the Lebesgue integral

#### Text Books:

1. InderK.Rana, *An Introduction to Measure Theory and Integration*, (2e), Narosa (2007)
2. H.L. Royden, P.M.Fitzpatrick, *Real Analysis –Fourth Edition*, Prentice Hall of India (2013)

#### Reference books:

1. De Barra.G, *Measure Theory and Integration*, 2e, New Age International Publishers (2013).
2. Howard J.Wilcox, *An Introduction to Lebesgue Integration and Fourier Series*, Dover (1995).
3. Paul R. Halmos, *Measure Theory*, Springer (1976).
4. N.L.Carothers, *Real Analysis*, Cambridge University Press( 2000).
5. C.D. Aliprantis and O.Burkinshaw, *Principles of Real Analysis*, 3e, Academic Press(Elsevier).
6. Edwin Hewitt and Karl Stromberg, *Real and Abstract Analysis*, Springer, 1975.





**M. Sc. MATHEMATICS**  
**HARD CORE- MATH-425: REAL ANALYSIS – II (4 credits)**

	<b>Course outcome</b>	<b>Level</b>
<b>CO 1</b>	understand the concepts of Riemann-Stieltjes integrals, and their properties.	Remember Understand
<b>CO 2</b>	solve problems using the Gauss, Stokes, and Divergence theorems	Apply
<b>CO 3</b>	examine the relations among the partial derivatives and total derivative, interchanging the order of the derivatives, interchanging the order of integrations.	Analyze
<b>CO 4</b>	discuss the proofs of Green's theorem, Stoke's theorem and Gauss divergence theorem.	Evaluate
<b>CO 5</b>	find examples to explain the differences between point-wise and uniform convergence of sequences of functions, to know what are the properties that are preserved under uniform convergence.	Create

**Objectives:**

To study Sequence and Series of functions, Power Series, Functions of Several Variables.

**Unit-I** ( Chapter:6 and Sections: 8.20 to 8.23 and 8.26 of [2] ) : Functions of bounded variation- Double sequences-Double series-Rearrangement theorem for double series-A sufficient condition for the equality of iterated series - Infinite products.

**Unit-II** (Chapter: 7 of [1], Subsections 7.1 to 7.25 ) : Sequence and Series of functions - Examples - Uniform convergence and Continuity - Uniform convergence and Integration - Uniform convergence and Differentiation - Double sequences and series - Iterated limits- Equicontinuous Families of Functions - Arzela – Ascoli Theorem

**Unit- III** (Chapter: 7 of [1] subsections: 7.26 to 7.33 and chapter 8 of [1]) : The Weierstrass theorem for algebraic polynomials- The Stone - Weierstrass Theorem - Power Series - The Exponential and Logarithmic Functions - The Trigonometric Functions - Fourier Series - The Weierstrass theorem for the Trigonometric polynomials.

**Unit- IV** (Chapter:9 of [1], Subsections: 9.6 to 9.23) : Functions of Several Variables - Linear Transformation - Differentiation - The Contraction Principle.

**Unit- V** (Chapter: 9 of [1], Subsections:9.24 to 9.38) : The inverse function Theorem - The implicit Function Theorem - The Rank Theorem – Determinants.

**Text Books**

1. Walter Rudin, Principles of Mathematical Analysis- McGraw Hill International Editions, Mathematics series, 1976.
2. Apostol, Mathematical Analysis, Narosa Publishing House, Indian edition,2002.

**Reference Books**

1. Patrick M. Fitzpatrick Advanced Calculus, Amer. Math. Soc. Pure and Applied Undergraduate Texts, Indian Edition, 2009.
2. Kenneth A. Ross, Elementary Analysis, The Theory of Calculus, Springer-Verlag,1980.
3. N.L.Carothers, *Real Analysis*, Cambridge University Press( 2000)
4. G.F.Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 2017.

**M.Sc MATHEMATICS**  
**HARD CORE: MATH 513 - FUNCTIONAL ANALYSIS**

	<b>Course Outcome</b>	<b>Level</b>
<b>CO 1</b>	explain the concepts of normed linear space (NLS), continuity of a linear map, $L_p$ -space, Banach, Hilbert spaces, four pillars	Remember & Understand
<b>CO 2</b>	demonstrate the convergence in the different types of spaces	Apply
<b>CO 3</b>	analyze the properties of different types NLS	Analyze
<b>CO 4</b>	determine the linear functional in terms orthonormal basis	Evaluate
<b>CO 5</b>	Obtain the open mapping theorem from closed graph theorem and vice-versa	Create

**Objectives:**

To study Normed Linear Spaces, Continuity, Equivalent norms, Hahn-Banach theorem for real vector spaces, Closed and open maps, Separable Hilbert spaces, Orthogonal projections.

**Unit-I** (Sections: 1.2.3, 1.2.5, 2.1, 2.1.1, 2.1.2, 2.1.4) : Review of linear spaces – Linear functionals – hyperspaces – projections – Cauchy Schwarz inequality – Holder’s inequality – Minkowski inequality – Normed linear spaces – Definition and examples – Basic properties – Semi norms and quotient spaces – product spaces and the graph norm.

**Unit-II** (Sections: 3.1, 3.1.1, 3.2, 3.2.1, 3.4.1, 2.2, 2.2.1, 2.2.2, 2.2.3, 2.4) : Bounded linear Maps – Properties – Norm of a bounded linear Map – Banach spaces – Completeness of  $l_p$  ( $1 \leq p \leq \infty$ ),  $L_p[a, b]$ ,  $C[a, b]$ ,  $BV[a, b]$  – Completeness of the space of all bounded linear Maps – The completeness of the quotient space – The completion of a normed linear space – Completeness and absolutely convergent series – Finite dimensional normed linear spaces – Riesz Lemma.

**Unit-III** (Sections: 5.1, 5.2, 5.3, 5.4, 3.4, 6.1) : The Hahn – Banach Extension Theorem and its corollaries – The Hahn Banach Separation Theorem – Convergence of sequence of operators – The uniform Boundedness principle – The Banach Steinhaus Theorem – Weakly bounded sets – Schauder basis and separability.

**Unit-IV** (Sections: 7.1, 7.2, 7.3, 8.1, 8.1.2) : The closed graph Theorem – The bounded inverse theorem – The open mapping Theorem – The dual of  $l_p$  ( $1 \leq p < \infty$ ), the dual of  $(C_{00}, \|\cdot\|_p)$  when  $(1 \leq p < \infty)$  - The dual of  $(C, \|\cdot\|_\infty)$ .

**Unit-V** (Sections: 2.1.5, 4.1, 4.2, 4.3, 4.4, 2.5, 2.6, 3.3) : Inner product spaces – Orthonormal sets – Gram Schmidt Orthogonalization process – Bessel’s inequality – Hilbert spaces - Parseval’s Theorem – Example of a nonseparable Hilbert space – Best approximation Theorems – Projection Theorem – Riesz Fischer Theorem – The Riesz representation Theorem.

**Text Book:** M.Thamban Nair, *Functional Analysis: A First Course*, Prentice Hall of India, 2002.

**Reference Books:**

1. Joseph Muscat, *Functional Analysis*, Springer(2008).
2. Balmohan V.Limaye, *Functional Analysis*, New Age International Publishers (2014).
3. Erwin Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley(2007).
4. Martin Schecter, *Principle of Functional Analysis*, American Mathematical Society (2009)
5. Bela Bollobas, *Linear Analysis: An Introductory Course, 2e*, Cambridge Univ. Press (1999).
6. Bryan P. Rynne and Martin A Youngson, *Linear Functional Analysis*, Springer (2008).

## M. Sc. MATHEMATICS

### HARD CORE: MATH-516 Theory of Numbers (4 Credits)

#### Objectives:

This course aims to explore primes, Power residues, Quadratic reciprocity, functions of number theory.

**UNIT-I :** Divisibility: Introduction -Divisibility- Primes.

**UNIT-II:** Solution of congruences – Congruences of higher degree – prime power moduli.

**UNIT-III:** Quadratic Residues, Quadratic reciprocity law, Jacobi Symbol.

**UNIT-IV:** Arithmetic functions-Recurrence functions, Mobious Inversion Formula, Irrational numbers, Irrationality of nth root of N, e and pi.

**UNIT-V:** Continued fractions and its convergence, representation of an irrational number by an infinite continued fraction. Some special quadratic surds.

#### Text Book

**Treatment as in I. Niven, H.S. Zuckerman and H.L. Montgomery** – An Introduction to the Theory of Numbers, New York, John Wiley and Sons, Inc., 2004, 5<sup>th</sup> Ed.

Unit I Section : 1.1-1.3

Unit II Section: 2.1-2.11

Unit III Section :3.1-3.3

Unit IV Section :4.1-4.3

Unit V Section: 5.6-5.11

#### Books for Reference:

1. **T.M. Apostol** – Introduction to Analytic Number Theory, Narosa Publishing House, New Delhi.
2. **G.H. Hardy and E.M. Wright**- An Introduction to the Theory of Numbers, Oxford University Press, 1979, 5<sup>th</sup> Ed.

**M.Sc. MATHEMATICS**  
**HARD CORE- MATH- 519- GALOIS THEORY (4 credits)**

**Objectives:**

To study Polynomial rings, Field theory, Splitting fields and Algebraic closures, Galois Theory and Composite extension and simple extensions.

**Unit I:** Field theory: Basic theory of field extensions-Algebraic Extensions.

**Unit II:** Splitting fields and Algebraic closures - Separable and inseparable extensions - Cyclotomic polynomials and extensions.

**Unit III:** Galois Theory: Basic definitions- The fundamental theorem of Galois Theory - Solvable by radicals.

**Unit IV:** - Galois groups over the rationales. Finite Fields- Wedderburn's theorem(First proof only)

**Unit V:** Classical straightedge and compass constructions, Cyclotomic extensions and Abelian extensions, Galois group of polynomials.

**Text Book:**

1. Abstract Algebra (Second Edition) by David S. Dummit and Richard M. Foote, Wiley Student Edition (1999) for Units I to III, (Chapter 13), (Sections 14.1 to 14.3 and 14.5-14.6).
2. Topics in Algebra by I.N. Herstein (Section 5.6 - 5.8), and (Section 7.1 -7.2).

**Reference books**

1. M. Artin: Algebra, Prentice-Hall of India, 1991.
2. N.Jacobson: Basic Algebra, Volumes I & II, W.H.Freeman, 1980.
3. S.Lang: Algebra, 3rd edition, Addison-Wesley, 1993.

## M. Sc. MATHEMATICS

### HARD CORE – MATH-537: PARTIAL DIFFERENTIAL EQUATIONS – (4 Credits)

	Course Outcome	Level
<b>CO 1</b>	understand the relation between the theory and modelling in the problems arising in various fields, such as, economics, finance, applied sciences and etc	Remember Understand
<b>CO 2</b>	Enhance their mathematical understanding in representing solutions of partial differential equations.	Apply
<b>CO 3</b>	classify the partial differential equations and transform into canonical form	Analyze
<b>CO 4</b>	determine the solution representation for the three important classes of PDEs, such as Laplace, Heat and wave equation by various methods.	Evaluate
<b>CO 5</b>	Formulate fundamentals of partial differential equations, like Green's function, maximum principles, Cauchy problem, to take a research career in the area of partial differential equations	Create

#### Objectives:

To study first order PDEs, Non-linear first order PDEs, Classification of second order PDEs, Wave Equations, Laplace equations, Heat Equations.

**Unit – I: First Order PDEs :** Surfaces and their Normals, Curves and tangents - Genesis of first order PDE- Classification of Integrals- Linear equations of first Order - Integral surface passing through a curve – Cauchy problem for first order PDE – Orthogonal Surfaces- Non-linear first order PDEs : Compatible systems- Solutions of Quasi linear equations- Charpit's method- Special Types of Charpits Method, -Integral surfaces through a given curve-The Cauchy problem for Quasi Linear case and nonlinear first order PDEs

**Unit – II: Second Order PDEs :** Genesis of Second order PDEs- Classification of second order PDEs- Canonical forms of Hyperbolic- Elliptic and parabolic type PDEs, Linear PDE with constant coefficients – Method of finding CF and particular integral- Homogeneous linear PDE

**Unit – III Hyperbolic PDEs / Wave Equation :** Derivation of One –dimensional wave equations- Initial Value Problem – D'Alembert Solution, Method of separation of variables, Forced Vibration, Solution of non-homogeneous equation Uniqueness of solution of wave equation.

**Unit – IV: Elliptic PDEs/Laplace Equations:** Derivation of Laplace equations & poisson equation- Boundary value problems- Properties of Harmonic functions- Spherical Mean, Mean value theorem- Maximum and minimum principles- Separation of variables- Dirichlet problem and Neumann problems for a rectangle and circle (Upto 2.10 in Text Book 1). Application - Irrotational Flow of an Incompressible Fluid (Section 2.13)

**Unit – V: Heat Equations:** Diffusion Equation, Boundary Conditions - Elementary solution- Solution by separation of variables- Classification in n-variables- Families of equipotential surfaces.

#### Text Books

1. K. Shankara Rao, Introduction to Partial Differential Equations, PHI Publications, 3<sup>rd</sup> Edition. 2011.
2. T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Publishing House, 2010.

#### Reference Books

1. I. N. Sneddon, Elements of Partial Differential Equations, McGraw Hill, International Edition, 1986.
2. F. John, Partial Differential Equations, Springer Verlag, 1975.
3. Lawrence C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, 1998.



**M.Sc. MATHEMATICS**  
**SOFT CORE: MATH-413 DISCRETE MATHEMATICS – (4 Credits)**

**Objectives:**

This course aims to explore Posets, Lattices, Boolean Lattices, Boolean Algebra, Boolean expressions, Logic gates, Karnaugh maps, Directed Graphs.

**Unit- I:**

Posets and lattices - Lattices as partially ordered sets – Properties of lattices - Lattices as algebraic systems – Sub lattices – Direct product – Homomorphism.

**Unit – II:**

Special lattices (Complete lattices, Bounded lattices, Complemented lattices, Distributive lattices, Modular lattices) and their properties – Boolean algebra – Switching algebra – Sub algebra – Direct product of Boolean algebra – Boolean homomorphism.

**Unit - III:**

Join irreducible elements – Atoms – Stone theorem – Boolean forms and their equivalence – Min terms – Sum of products canonical form – Free Boolean algebra – Max terms and product of sums canonical form – Values of Boolean expressions – Boolean functions – Symmetric Boolean expressions.

**Unit – IV:**

Logic gates – Combination of gates – Adders – Karnaugh maps – Representation and Minimization of Boolean functions.

**Unit –V:**

Directed graphs – In and out degrees – Connectedness - – Directed paths and cycles – Moon theorem.

**Text Books**

1. J.P Trembly and R. Manohar: Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw – Hill Publishing Company Ltd, New Delhi 1997.  
For Units– I, II, III, IV: Relevant portions of Chapter - 4.
2. J.A. Bondy and U.S.R. Murthy: Graph Theory with Applications, Macmillan Press Ltd, New Delhi-1976.  
For Unit –V: Relevant portions of Chapter 10.

**Reference Books**

1. J. Johnsonbaugh, Discrete Mathematics, MacMillan Publishing company, New York 1989.
2. R.P. Grimaldi, Discrete and Combinatorial Mathematics (An Applied Introduction), Pearson Edition Asia, New Delhi – 2002.
3. B. Kolman, R.C. Busby and S.C Ross, Discrete Mathematical Structures, Pearson Editionn Pvt Ltd, New Delhi –2003.

**M. Sc. Mathematics**  
**SOFT CORE- MATH-415: CONTINUUM MECHANICS (4 Credits)**

**Objectives:**

To study the basics of Vectors, Equations of equilibrium, Analysis of deformation and Derivation of field equations.

**UNIT I**

Vectors – summation convention – translation and rotation of coordinates – coordinate transformations in general – analytical definitions of scalars, vectors, and Cartesian tensors

**UNIT II**

Stress components – Cauchy's formula – equations of equilibrium – change of stress components in transformation of coordinates – stress components in orthogonal curvilinear coordinates – stress boundary conditions.

**UNIT III**

Plane rate of stress – Mohr's circle – principal stresses – shearing stresses - stress deviation tensor – Lamé's stress ellipsoid.

**UNIT IV**

Analysis of deformation – strain – strain components in rectangular Cartesian coordinates – infinitesimal rotation – finite strain components.

**UNIT V**

Derivation of field equations – material description of the motion of a continuum – spatial description – equation of motion – equation of motion - moment of momentum – balance of energy.

**Text Book:**

1. Y.C.FUNG, "A First Course in Continuum Mechanics" Second Edition, Prentice Hall.

**Reference Book:**

1. L.A.Segel and G.H.Handelman Mathematics Applied to Continuum Mechanics SIAM (2007)

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-416 GRAPH THEORY (4 Credits)**

**Objectives:**

To study basic of Graphs and Trees, Counting the Number of Spanning Trees, Matchings, Chromatic Number and Planarity of Graphs.

**Unit -I**

Graphs – Subgraphs – Isomorphism of graphs – Degrees of Vertices – Paths and Connectedness – Automorphism of a Simple Graph – Operations on Graphs – Trees – Centers and Centroid.

**Unit -II**

Counting the Number of Spanning Trees – Cayley's Formula – Vertex Cuts and Edge Cuts – Connectivity and Edge-connectivity – Blocks – Cyclical Edge-connectivity of a Graph.

**Unit -III**

Vertex Independent sets and Vertex Coverings – Edge-Independent Sets – Matchings and Factors – M-Augmenting Paths – Matchings in Bipartite Graphs – Halls Theorem on Bipartite graphs – Tutte's 1-Factor Theorem.

**Unit -IV**

Vertex Coloring – Chromatic Number – Critical Graphs – Brooks' Theorem – Girth – Triangle-Free Graphs – Mycielski's Construction – Edge Colorings of Graphs – Vizing's Theorem – Chromatic Polynomials.

**Unit -V**

Planar and Nonplanar Graphs – Euler's Formula and its Consequences –  $K_5$  and  $K_{3,3}$  are Nonplanar graphs – Dual of a Plane Graph – The Four Color Theorem and the Heawood Five-Color Theorem – Kuratowski's Theorem (without proof).

**Text Book:-**

1. R. Balakrishnan and K. Ranganathan, A Textbook of Graph Theory (Universitext), Second Edition, Springer New York 2012.

Chapter 1: 1.1-1.6, 1.8

Chapter 3: 3.1-3.5

Chapter 4: 4.1-4.5

Chapter 5: 5.1-5.5

Chapter 7: 7.1,7.2,7.3.1, 7.5,7.6.2,7.9

Chapter 8: 8.1-8.7

**Reference Books:-**

1. J.A. Bondy and U.S.R. Murty, Graph Theory, Springer 2008.
2. Douglas B. West, Introduction to Graph Theory, Second Edition, PHI Learning Private Ltd, New Delhi-2011.
3. G. Chartrand, Linda Lesniak and Ping Zhang, Graphs and Digraphs, Fifth Edition, CRC press 2011.

**M.Sc. MATHEMATICS**  
**SOFT CORE: MATH-417 SCILAB PRACTICALS (4 Credits)**

**Unit-I: SCILAB Basics:** Overview of SCILAB – Get started – Basic elements of the language – Matrices

**Unit-II: SCILAB Programming:** Looping and Branching – Functions – Plotting

**Unit-III: Application into Basic Statistics:** Measures of Central Tendency – Descriptive Statistics – Measures of Dispersion

**Unit-IV: Application into Advanced Statistics,** Cumulated distribution function – Data with missing values – Hypothesis Testing

**Unit-V: Optimization Toolbox:** FOSSEE SCILAB Optimization Toolbox – fminsearch – fsolve – fminbnd – fmincon – linprog – intlinprog – intfmincon – quadprog – intquadprog, Genetic Algorithm (optim\_ga)

Unit-I: Chapter 1, 2, 3, 4 of Text Book 1

Unit-II: Chapter 5, 6, 7 of Text Book 1

Unit-III: Text Book 2

Unit-IV: Text Book 2

Unit-V: Text Book 3 and 4

Text Books:

1. Introduction to SCILAB – Michael Baudin From SCILAB Consortium, 2010
2. SCILAB Online Help –  
[https://help.scilab.org/docs/5.5.2/en\\_US/section\\_33491857221a48388b878311e9f4b67e.html](https://help.scilab.org/docs/5.5.2/en_US/section_33491857221a48388b878311e9f4b67e.html)
3. FOSSEE SCILAB Toolbox – Optimization Toolbox <https://scilab.in/fossee-scilabtoolbox/optimization-toolbox/functions>
4. SCILAB Online Help – optim\_ga –  
[https://help.scilab.org/docs/5.5.2/en\\_US/optim\\_ga.html](https://help.scilab.org/docs/5.5.2/en_US/optim_ga.html).

Reference Books:

1. <https://www.scilab.org/tutorials>
2. SCILAB help documentation – Statistics
3. Basic Statistics and Probability with SCILAB – Gilberto E. Urroz infoclearinghouse.com  
<https://www.scilab.org/sites/default/files/Basic%20Statistics%20and%20Probability%20with%20SCILAB%20-%20Gilberto%20E.%20Urroz%20-%202001.pdf>

**Objectives:**

To study the basics of SCILAB operations for Matrices, Looping, Plotting of graphs and Solving Ordinary Differential Equations.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-511 FLUID MECHANICS (4 Credits)**

**Objectives:**

This course aims to explore Conservation of mass and energy, Rotations and vorticity, Stokes equations, Potential flow, Boundary layers

**Unit-I**

Equations of motion - Euler's Equation – Conservation of mass – Balance of momentum – Transport theorem - Conservation of energy – Incompressible Flows – Isentropic Fluids – Bernoulli's theorem.

**Unit-II**

Rotations and vorticity – Kelvin's circulation theorem – Helmholtz's theorem.

**Unit-III**

Navier- Stokes equations – Scaling properties – Decomposition theorem - Stokes equations – Poiseuille flow .

**Unit-IV**

Potential flow – Complex potential – Blasius theorem - Kutta-Joukowski theorem – D'Alembert's paradox – Stokes paradox.

**Unit-V**

Boundary layers – Prandtl boundary layer equations –Steady boundary layer flow on a flat plate of infinite width.

**Text Book**

A. J. Chorin and J. E Marsden, A Mathematical Introduction to Fluid Mechanics, Texts in Applied Mathematics 4, Springer Verlag, 1990.

**References:**

1. D.J.Acheson, Elementary Fluid Dynamics, Oxford University Press, 1990.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH 512: DIFFERENTIAL GEOMETRY ( 4 Credits )**

**Objectives:**

This course aims to explore Curves, Torsion, surface, Isometrics of surfaces, Curvature of curves on a surface.

**Unit-I**

Curves- arc length- Repara metrization -Level curves - Curvature - Plane curves.

[ Sections: 1.1 to 1.4 and Sections 2.1,2.2.]

**Unit-II**

Space curves-Torsion- Serret Frenet equations- Simple closed curves- The Isoperimetric Inequality- The Four vertex Theorem.

[Sections 2.3 and Sections 3.1 to 3.3.]

**Unit-III**

Smooth surface- Tangents, normal and orientability- Examples of surfaces- Quadratic surfaces- Triple orthogonal systems- Applications of Inverse function theorem.[Sections 4.1 to 4.7]

**Unit-IV**

Lengths of curves on surfacesa- First fundamental form- Isometries of surfaces- Conformal mapping of surfaces-Surface area- Equiareal maps and a theorem of Archimedes.

[ Sections: 5.1 to 5.5]

**Unit-V**

The Second Fundamental form- The Curvature of curves on a surface- The normal and principal curvature- Euler's theorem- The geometric interpretation of principal curvatures.

[ Sections: 6.1 to 6.4]

**Text Book:**

1. Andrew Pressley, *Elementary Differential Geometry*, Springer, 2004.

**Reference Books:**

1. Christian Bar, *Elementary Differential Geometry*, Cambridge University Press, 2011.
2. Thomas F. Banchoff and Stephen T. Lovett, *Differential Geometry of Curves and Surfaces*, A.K Peters/CRC press, 2010.
3. W. Klingenberg, *A course in Differential Geometry*, Springer-Verlag, New York, 1978.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-514 ANALYTICAL DYNAMICS ( 4 Credits)**

**Objectives:**

To study the basics of Generalized coordinates, Lagrange equation for non-holonomic constraints, Hamilton's equation and Harmonic oscillator.

**Unit- I**

Generalized coordinates-Virtual displacements-D'Alembert's principle and derivation of the Lagrange equations.

**Unit -II**

Lagrange equation for non-holonomic constraints-Method of Lagrange multipliers-Velocity dependent potentials-Non-conservative forces and dissipation function- Non-holonomic systems and Lagrange multipliers.

**Unit- III**

Hamilton's equation-The Hamilton principle - Variational principle- Catenary- Brachistochrone-Derivation of the Hamiltonian equation- Liouville's theorem.

**Unit- IV**

Canonical transformations – Generating functions- Harmonic Oscillator.

**Unit- V**

Hamilton – Jacobi theory- Harmonic oscillator- action – Angle variables- Kepler problem- particle motion in a potential with azimuthal symmetry – Slant throw.

**Text Book**

Walter Greiner, Classical Mechanics, Systems of Particles and Hamiltonian Dynamics, Springer, ISE, 2004.

Unit I: Sections 14-15; Unit II: Sections 16-17; Unit III: Sections 18 ; Unit IV: Sections 19;

Unit V: Sections 20.

**Reference Books**

1. H. Goldstein, Classical Mechanics, Narosa Publishing House, New Delhi, 1985.
2. F. Scheck, Mechanics, From Newton's Laws to Deterministic Chaos, Springer, 1999.

**M. Sc. Mathematics**  
**SOFT CORE: MATH-515: FUZZY SETS AND ITS APPLICATIONS**  
**(4 Credits)**

**Objectives:**

To study the basics of Crisp sets, Fuzzy relation, Fuzzy control structure and Fuzzy Decision making.

**Unit-I: Crisp Sets**

Basic Definitions - Operations on crisp sets – Properties of crisp set – Crisp relations- Operations on crisp relations – Properties of Crisp relations – Composition of Crisp relations - Characteristic Function-Exercises

**Unit – II: Fuzzy Sets**

Definition of Fuzzy sets - examples - Fuzzy numbers- Characteristics of a Fuzzy Set- Basic operations on fuzzy sets- Properties of Fuzzy sets- Membership functions-Algebraic product and Sum of Fuzzy Sets – Power and related operations on Fuzzy Sets – The extension Principle-Exercise

**Unit-III: Fuzzy Relations**

Definition of Fuzzy relation – Basic operations on Fuzzy relations – Direct product – Projections of a Fuzzy relation – Max-Min and Min-Max compositions – Fuzzy relations and approximate Reasoning – Exercise- Fuzzy relational equation-Problem partitioning – Solution method – Use of Neural network in Fuzzy relation

**Unit-IV: Fuzzy control systems**

Introduction – Fuzzy control structure - Modelling and control parameters – If....and....then rules – Rule evaluation – Conflict resolution – Defuzzification – Fuzzy controller with matrix Representation - Exercises.

**Unit-V: Applications**

Fuzzy Control in Washing Machine – Fuzzy Decision making in forecasting – Fuzzy decision Making in industrial problems – Fuzzy control in traffic control – Fuzzy relational equation in medicine.

**Text Books**

1. George J. Klir/Bo Yuan, Fuzzy Sets and Fuzzy Logic, Prentice Hall of India (2000).
2. George Bojadziev and Maria Bojadziev, Fuzzy Sets, Fuzzy Logic, Applications, World Scientific Publishing Co.Pte.Ltd, Singapore, 1996.

**Reference Books**

1. George J. Klir and Tina A. Folger, Fuzzy Sets, Uncertainty and Information, Prentice-Hall of India 1993.
2. Witold Pedrycz & Fernando Gomide, An introduction to Fuzzy Set, Prentice-Hall of India, New Delhi.2005.
3. James J. Buckley, Esfandiar Eslami, An introduction to Fuzzy Logic and Fuzzy Sets, Springer 2002.
4. Abraham Kandel and Gideon Langholz, Fuzzy Control Systems, CRC Press, USA 1994.



## M. Sc. MATHEMATICS

### SOFT CORE : MATH-522 ALGEBRAIC GRAPH THEORY (4 Credits)

#### Objectives:

To study the objectives of Linear Algebra in Graph Theory, Spanning Trees and Associated Structures, The Multiplicative Expansion and Chromatic Polynomial.

**Unit -I:** Linear Algebra in Graph Theory – The Spectrum of a Graph – Characteristic polynomial – Adjacency Algebra - Reduction Formula for  $\chi$  – Regular Graphs and Line Graphs – Circulant Graph – Spectrum of the Strongly Regular Graph – Cycles and Cuts – The Incidence Matrix – The Laplacian Spectrum.

**Unit -II:** Spanning Trees and Associated Structures – Kirchhoff's Law – Thomson's Principle – The Tree-Number – A Bound for the Tree Number of Regular Graphs – Determinant Expansions – Elementary Graphs.

**Unit -III:** Vertex-Partition and the Spectrum – Color Partition – Wilf's Theorem on the Chromatic Number of a Graph – Coloring Problems – The Chromatic Polynomial – Recursive Relation for the Chromatic Polynomial – Quasi-Separable Graphs – Subgraph Expansions – The Rank Polynomial.

**Unit -IV:** The Multiplicative Expansion – Whitney's Theorem on Counting Subgraphs – The Induced Subgraph Expansion – Baker's Theorem.

**Unit -V:** The Tutte Polynomial – The  $\lambda$ -operator – The Deletion-Contraction Property – Chromatic Polynomial and Spanning Trees – The Chromatic Invariant.

#### Text Book:-

1. Norman Biggs, Algebraic Graph Theory, Second Edition, Cambridge University Press, 1993.

#### Reference Books:-

1. Chris Godsil and Gordon Royle, Algebraic Graph Theory, Springer 2009.
2. R. Balakrishnan and K. Ranganathan, A Textbook of Graph Theory (Universitext), Second Edition, Springer New York 2012.

**M.Sc. MATHEMATICS**  
**SOFT CORE: MATH - 523 -ALGORITHMIC GRAPH THEORY – (4 Credits)**

**Objectives:**

To study Graphs and notations, Spanning trees and Search methods, finding all spanning trees of a graph, matrix in graphs and algorithm for finding spanning tree.

**Unit- I**

Graphs and notations – Null, complement and complete graphs – Degrees – Isomorphism – Subgraphs – Paths and cycles – Connectedness - Components – Operations on graphs - Distance, Radius, Diameter, Centers and medians in graphs - Rooted and m ary trees.

**Unit – II**

Binary trees – and their search – Spanning trees - Search methods: Breadth first search and depth first search algorithms – Introduction to groups, fields and vector spaces – Vector spaces of graphs – Dimensions, Relationships, orthogonality of cycle and cutset subspaces.

**Unit – III**

Fundamental cycles – Finding all spanning trees of a graph - Cutsets and their properties – Fundamental cutsets – Relation in fundamental cycles and cutsets – On Connectivity and separability – Network flows - (1) isomorphism.

**Unit – IV**

Incidence matrix and its sub matrices – Cycle matrix – Fundamental cycle matrix and its rank and nullity – Cutset and fundamental cut set matrices – Relationship theorem – Path matrix – Adjacency matrix.

**Unit –V**

The connector problem – Kruskal algorithm – Prim algorithm – The shortest path problem – Dijkstra algorithm – Network models – Flows – cuts – Maximum flow algorithm – The max. flow, min. cut thorem.

**Text Books**

1. K. Thulasiraman and M.N.S. Swamy, Graphs : Theory and Algorithms – John Wiley and Sons, Inc., New York (1992).
2. Narsingh Deo, Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India Ltd (1974).
3. J. A. Bondy and U.S.R. Murthy: Graph Theory with Applications, Elsevier Science North Holland (1982).

**Reference Books**

1. G. Chartrand and O. R. Oellermann, Applied and Algorithmic Graph Theory, McGraw Hill New York (1993).
2. W. Kocay and D. L. Kreher, Graphs, Algorithms and Optimization, Chapman and Hall – CRC Press, London (2005).
3. J. Johnsonbaugh, Discrete Mathematics, Macmillan Publishing Company, New York (1989).

**M. Sc. MATHEMATICS**  
**SOFT CORE : MATH-527 ALGEBRAIC NUMBER THEORY (4 Credits)**

**Objectives:**

To study the basics of Primes, Principal ideal domain, Euclidean domain, Algebraic number and The Norm and the Trace.

**Unit-I Elementary Number Theory**

Integers – Greatest common divisor – Infinitude of primes – Unique factorization in  $\mathbb{Z}$  – Fermat's little theorem – Euler's  $\Phi$  function and Euler's theorem – Multiplicative property of  $\Phi$  function – Applications of unique factorization – The equation  $x^2 + y^2 = z^2$  – The equation  $x^4 + y^4 = z^2$  – The equation  $x^4 - y^4 = z^2$  – Fermat numbers and their properties.

**Unit-II Euclidean Rings**

Preliminaries: Units, Associates, Irreducible elements, Norm map, Unique factorization domain, Principal ideal domain, Euclidean domain – Gauss' lemma – Gaussian integers – Units and primes in the ring of Gaussian integers – Eisenstein integers – Units in the ring of Eisenstein integers – Factorization of 3 – Order of  $\mathbb{Z}[\rho]/(\lambda)$ .

**Unit-III Algebraic Numbers and Integers**

Basic concepts – Algebraic number – Algebraic integer – Minimal polynomial  
Countability of algebraic numbers – Liouville's theorem for  $\mathbb{R}$  – Algebraic number fields – Theorem of the primitive element – Liouville's theorem for  $\mathbb{C}$  – Characterization of algebraic integers.

**Unit-IV Integral Bases**

The norm and the trace – Integral basis for an algebraic number field – Algebraic integers of  $\mathbb{Q}(\sqrt{-5})$  – Existence of an integral basis – Discriminant of an algebraic number field – Index – Determination of an integral basis for the ring of integers of a quadratic number field.

**Unit-V Dedekind Domains**

Integral closure – Integrally closed ring – Noetherian ring – Dedekind domain – Characterizing Dedekind domains.

**Text Book**

J.E.Smonde and M.Ram Murty, Problems in Algebraic Number Theory,  
Graduate Texts in Mathematics, Volume 190, Springer Verlag, New York, 1999.

Sections 1.1 and 1.2

Sections 2.1, 2.2 and 2.3

Sections 3.1, 3.2 and 3.3

Sections 4.1, 4.2 and 4.3

Sections 5.1 and 5.2

**Reference Books:**

1. Pierre Samuel and Allan J Silberger, Algebraic Theory of Numbers, Dover Pub. Inc, 2008.

## M. Sc. MATHEMATICS

### SOFT CORE: MATH-528 ADVANCED ALGEBRAIC NUMBER THEORY( 4 Credits )

#### Objectives:

To study the basics of Euclidean rings, Quadratic residues and quadratic non-residues, Units in real quadratic fields and The Riemann and Dedekind zeta functions.

#### Unit-I The Ideal Class Group

Euclidean rings – Hurwitz constant – Fractional ideals – Finiteness of the ideal class group – The class number of an algebraic number field – The class number of  $\mathbb{Q}(\sqrt{-5})$  – The Diophantine equation  $x^2 + 5 = y^3$ .

#### Unit-II Quadratic Reciprocity

Preliminaries – Quadratic residues and quadratic non residues – The Legendre symbol – The quadratic character of -1 and 2 – Gauss sums – The law of quadratic reciprocity.

#### Unit-III The Structure of Units

Discrete subgroup of  $\mathbb{R}^m$  – Dirichlet's unit theorem – Units in real quadratic fields – Pell's equation.

#### Unit-IV Higher Reciprocity Laws

Cubic reciprocity – Eisenstein reciprocity.

#### Unit-V Analytic Methods

The Riemann and Dedekind zeta functions – Zeta functions of quadratic fields – Dirichlet's hyperbola method.

#### Text Book

J. E. Smonde and M. Ram Murty, Problems in Algebraic Number Theory, Graduate Texts in Mathematics, Volume 190, Springer Verlag, New York, 1999.

Sections 6.1, 6.2 and 6.3

Sections 7.1, 7.2 and 7.3

Sections 8.1 and 8.2

Sections 9.1 and 9.2

Sections 10.1 and 10.2

#### Reference Books:

1. Pierre Samuel and Allan J Silberger, Algebraic Theory of Numbers, Dover Pub. Inc, 2008

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-529 THEORY OF FUZZY SETS (4 Credits)**

**Objectives:**

To study the basics of Fuzzy sets, Fuzzy complements, Fuzzy numbers, Crisp and fuzzy relations and Fuzzy propositions.

**Unit-I Fuzzy sets**

Basic definitions – Types of fuzzy sets – Basic concepts –  $\alpha$  cuts and their properties – Representations of fuzzy sets – first and second decomposition theorems.

**Unit-II Operations on fuzzy sets**

Types of operations – Fuzzy complements – Fuzzy intersections: t norms – Fuzzy unions: t conorms – Combinations of operations.

**Unit-III Elements of fuzzy arithmetic**

Fuzzy numbers – Linguistic variables – Arithmetic operations on intervals – Arithmetic operations on fuzzy numbers – Fuzzy equations.

**Unit-IV Fuzzy relations**

Crisp and fuzzy relations – Projections and cylindric extensions – Binary fuzzy relations – Binary relations on a single set – Fuzzy equivalence relations – Fuzzy compatibility relations.

**Unit-V Fuzzy logic**

An overview of classical logic – multi valued logics – Fuzzy propositions – Fuzzy quantifiers – Linguistic hedges – Inference from conditional fuzzy propositions.

**Text Book**

George J. Klir and Bo Yuan, Fuzzy sets and Fuzzy Logic, Theory and Applications, Prentice – Hall of India Private Limited, New Delhi, 2000.

**Reference Book**

H. J. Zimmermann, Fuzzy set theory and its Applications, Allied Publishers Limited, New Delhi, 1991.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH -531 CRYPTOGRAPHY ( 4 Credits)**

**Objectives:**

To study the basics of Public and Secret Key Encryption, Digital Signatures and Network Security.

**Unit I**

**Introduction:** Overview of course- Classical cryptography [parts of Chapter 1].

**Unit II**

**Secret Key Encryption :** Perfect Secrecy - One time pads [Chapter 2.1], Stream ciphers and the Data Encryption Standard (DES) [Chapter 3 (excluding 3.6)], The Advanced Encryption Standard (AES) - adopted September 2000.

**Unit III**

**Public Key Encryption :** Factoring and the RSA encryption [Chapter 4.1 -4.4], Discrete log- Diffie-Hellman Key Exchange [Chapter 8.4 (only pages 270-273)].

**Unit IV**

**ElGamal encryption** [Chapter 5 (only pages 162-164)] , Digital Signatures [Chapter 6 (excluding 6.5 - 6.6)], One-time signatures- Rabin and ElGamal signatures schemes- Digital Signature Standard (DSS).

**Unit V**

**Hashing :** Motivation and applications- Cryptographically Secure Hashing. [Chapter 7.1-7.3,7.6], Message Authentication Codes (MAC)- HMAC- Network Security - Secure Socket Layer (SSL)- IPsec., Secret Sharing- Definition. Shamir's threshold scheme [Chapter 11.1], Visual secret sharing schemes.

**Text Book**

D. R. Stinson, Cryptography, Theory and Practice, CRC Press, 1995.

**Reference Books**

1. Richard A. Mollin, An Introduction to Cryptography, Chapman & Hall / CRC, Boca Raton, 2000
2. Dominic Walsh, Codes and Cryptography, Oxford Science Publications, Clarendon Press, Oxford, 1988

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-533 ADVANCED TOPICS IN TOPOLOGY AND ANALYSIS**  
**(4 Credits)**

**Objectives:**

To study the basics of Quotient topology and quotient maps, One point compactification, Local finiteness and Stricly convex spaces.

**Unit-I**

Quotient topology and quotient maps - Examples of quotient spaces - Path connectedness - Standard results - Example of a connected but not path connected space- Locally connected spaces.

**Unit-II**

The Uryshon's metrization theorem - One point compactification - Stone- Cech compactification - The Arzela - Ascoli theorem.

**Unit-III**

Local finiteness- Countably locally finite refinement of open coverings of metric spaces – Paracompactness - Standard results - Metric spaces are paracompact.

**Unit-IV**

Partition of unity -  $L_p$ - spaces – Completeness - Dual of  $L_p[a, b]$  for  $1 \leq p < \infty$ .

**Unit-V**

Extreme points - Caratheodory's theorem - Krein- Milman theorem - Milman converse Theorem - Extreme points of the closed unit ball of  $c$ ,  $l_\infty$ ,  $C(Q)$ ,  $Q$  compact, Hausdorff and the dual of  $C(Q)$  - Stricly convex spaces - Examples.

**Text Books:**

1. James R. Munkres, Topology by James R. Munkres, Pearson, 2<sup>nd</sup> edition, 2000.
2. H.L.Royden, and P.M.Fitzpatrick, Real Analysis, (Fourth Edition) PHI Learning Private Limited, 2011.
3. M. Fabian, P.Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizler: Functional Analysis and Infinite dimensional geometry, CMS Books in Mathematics, Springer-Verlag, 2001.

**Reference Books:**

1. James Dugundji, General Topology, Allyn and Bacon, Inc.(1966).
2. Joseph Conway, A course on Functional Analysis, Springer, 2<sup>nd</sup> edition, 1997.
3. B.V.Limaye, Functional Analysis, Wiley Eastern, New Delhi, 1981

**M. Sc. MATHEMATICS**  
**SOFT CORE : MATH-534 APPROXIMATION THEORY ( 4 Credits )**

**Objectives:**

To study the basics of Interpolation by polynomials, Approximation of continuous, periodic functions by trigonometric polynomials, Uniqueness of polynomials of best approximation and Approximation from finite dimensional subspaces.

**Unit-I**

Interpolation by polynomials - Lagrange interpolation - Vander Monde's determinant- Bernstein polynomials - Weierstrass approximation theorem.

**Unit-II**

Stone- Weierstrass theorem (Real and complex versions) - Weierstrass theorem as corollary - Approximation of continuous, periodic functions by trigonometric polynomials - Best approximation in  $C[a, b]$  with sup norm - Chebychev's Alternation theorem - Theorem of de La Vallee Poussin.

**Unit-III**

General linear families - Haar system and its characterizations - Uniqueness of polynomials of best approximation - Strong unicity theorem - Harr's unicity theorem.

**Unit-IV**

An algorithm of Remes and convergence under Haar condition - Strictly convex and uniformly convex Banach spaces - Approximation in inner product spaces – Approximation from closed, convex subsets - Approximation from subspaces of Hilbert spaces - Uniform convexity and continuity of metric projection.

**Unit-V**

Approximation from finite dimensional subspaces - Normal equations and Gram's determinant - approximation in  $L^2[a, b]$  - Orthogonal polynomials - Legendre and Chebychev polynomials.

Best approximation by subspaces of Banach spaces - Duality formula - Spaces in which all closed subspaces are proximal or Chebychev-proximality of weak\* Closed subspaces - Approximation by closed hyperplanes.

**Text Book:**

1. E.W.Cheney, An Introduction to Approximation theory, McGraw-Hill, 1<sup>st</sup> edition, 1966.

**Reference Books:**

1. B.V. Limaye, Functional Analysis, New Age International pvt. Ltd., 2008.
2. Frank Deutsch, Best approximation in inner product spaces, spinger,2001.
3. Serge Lang, Real Analysis, Addison- Wesley, 1983.
4. Ivan Singer, Best approximation in normed linear spaces by elements of linear Subspaces, Springer-Verlag, 1970.



**M. Sc. MATHEMATICS**  
**SOFT CORE : MATH-536 DIFFERENCE EQUATIONS (4 Credits)**

**Objectives:**

To study the basics of Difference Calculus, First order difference equation, General Linear equation, Linear Difference equations, Inhomogeneous equations, Linear Difference equation with constant coefficients.

**Unit-I**

The Difference Calculus Definition, Derivation of difference equation- Existence and uniqueness theorem- Operators and E- Elementary difference operators. Factorial polynomials- Operators and the sum calculus- Examples.

**Unit-II**

First order difference equation General Linear equation- Continued fraction. A general first-order equation – Expansion Techniques.

**Unit-III**

Linear Difference equations Introduction- Linearly dependent functions- Fundamental theorem for homogeneous equations.

**Unit-IV**

Inhomogeneous equations In homogeneous equations- Second order equations. Sturm Liouville Difference equations.

**Unit-V**

Linear Difference equation with constant coefficients introduction- Homogeneous equation- Construction of a difference equation having specified solution- Relationship between Linear difference and differential equation.

**Text Book**

Ronald E. Mickens, Difference equation - Theory and Application, Chapman & Hall, Second Edition, New York – London, 1990.

Unit 1: Chapter 1: (Sections 1.2-1.8)

Unit 2: Chapter 2: (Sections 2.1-2.8)

Unit 3: Chapter 3: (Sections 3.1-3.3)

Unit 4: Chapter 3: (Sections 3.5-3.6)

Unit 5: Chapter 4: (Sections 4.1-4.4)

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-538 LIE GROUPS OF TRANSFORMATIONS AND**  
**DIFFERENTIAL EQUATIONS ( 4 Credits )**

**Objectives:**

To study the basics of Lie groups of transformation, Lie Algebras and Invariance of partial differential equations of first and second order.

**Unit-I**

Introduction–Lie groups of transformation – Infinitesimal transformations.

**Unit-II**

Extended group transformations and infinitesimal transformations (one independent – one dependent and two independent – two dependent).

**Unit-III**

Lie Algebras and Applications.

**Unit-IV**

Invariance of first and second order differential equations.

**Unit-V**

Invariance of partial differential equations of first and second order – elementary examples.

**TextBook**

**Treatment as in** G. W. Bluman and S. Kumei, Symmetries and Differential Equations, Springer – Verlag , 1980.

Unit 1 – Chapter 2 (Sections 2.1 – 2.2);

Unit 2 – Chapter 2 (Sections 2.3.1 – 2.3.3) ;

Unit 3 – Chapter 2 (Sections 2.4.1 – 2.4.4);

Unit 4 – Chapter 3 (Sections 3.1.1 – 3.3.3);

Unit 5 – Chapter 4 (Sections 4.4.1 – 4.2.2).

**M. Sc. MATHEMATICS**  
**SOFT CORE : MATH-539 NUMERICAL ANALYSIS FOR ORDINARY**  
**DIFFERENTIAL EQUATIONS (4 Credits)**

**Objectives:**

To study various numerical methods to solve ordinary differential equations such as Euler's method, Gauss quadrature and Error Control.

**Unit-I**

Euler's method - Trapezoidal rule - Theta method.

**Unit-II**

Adams - Bashforth method - Order and convergence - Backward differentiation formula.

**Unit-III**

Gauss quadrature - Explicit Runge - Kutta scheme - Implicit Runge Kutta scheme - Collocation.

**Unit-IV**

Stiff equations - Linear stability domain and A- Stability -- A-stability of RK and multistep methods.

**Unit-V**

Error Control - Milne Device - Embedded Runge Kutta method.

**Text Book**

1. Arieh Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge University press, 2<sup>nd</sup> edition, 2008.

**Reference Books:**

1. Richard L. Burden and J. Douglas Faires, Numerical Analysis(9<sup>th</sup> Edition), Cengage Learning India, 2012.

**M. Sc. MATHEMATICS**  
**SOFT CORE : MATH-540 ADVANCED FLUID MECHANICS (4 Credits )**

**Objectives:**

This course aims to explore Wave equation, Prandtl's relation and Compressive shocks, Riemann problem, conservation laws and Non convex systems of conservation laws, Numerical methods.

**Unit-I**

Characteristics - Wave equation - Examples - Riemann invariants - Hodograph transformation - Piston problem.

**Unit-II**

Shocks - Systems of conservation laws - Weak solution - Rankine - Hugoniot relations - Hugoniot relation - Prandtl's relation - Compressive shocks - Entropy condition.

**Unit-III**

Riemann problem - Centered waves - Solution of the Riemann problem - Courant – Fricdricks - Lewy condition.

**Unit-IV**

Combustion waves - Single conservation law - Convex conservation laws - Oleinik's condition – Non convex systems of conservation laws - Solution.

**Unit-V**

Numerical methods - Finite difference Methods- Forward Difference - Backward Difference - Central difference - Consistency - Order - Stability - Lax's Theorem – Von Neumann Analysis - Godunov scheme -  $l_1$  stability -  $l_2$  stability - Lax – Fricdricks scheme - Lax Wendroff scheme - Crank - Nicolson scheme.

**Text Books**

1. Chorin and Marsden, A Mathematical Introduction to Fluid Mechanics, Texts in Applied Mathematics, Springer, Third Edition, 2009.
2. A Iserles , A First course in the Numerical Analysis of Differential Equations, Cambridge University Press, 2009.

**Reference Books:**

1. D.J.Acheson, Elementary Fluid Dynamics, Oxford University Press, 1990

**SOFT CORE: MATH-541 INTEGRAL EQUATIONS (4 Credits)**

**Course Objectives:**

The main objectives of this course are to:

1. Introduce the various types of integral equations.
2. Study the methods of successive approximations and Fredholm theory.
3. Acquire knowledge on applications to Ordinary Differential Equations.

**Course Outcomes:**

After successful completion of the course the student will be able to

**CO1** Define the methods to solve integral equations.

**CO2** Discuss the method of successive approximation and Fredholm theory.

**CO3** Identify and Construct the solutions for real time applications.

**Unit-I** Introduction – Definition – Regularity conditions – Special kinds of Kernels – Eigen values and Eigen functions – Convolution integral – The Inner or Scalar Product of Two Functions - Integral Equations with Separable Kernels - Reduction to a system of algebraic equations – Examples  
**Chapter1:** Sections 1.1 to 1.6 **Chapter2:** Sections 2.1 to 2.2

**Unit-II** Fredholm alternative–Examples – An approximation method - Fredholm Integral Equation of the First Kind - Method of successive approximations – Method Of Successive Approximations - Iterative scheme – Examples  
**Chapter2:** Sections 2.3 to 2.6 **Chapter3:** Sections 3.1 to 3.2

**Unit-III** Volterra integral equations – Examples – Some results about the resolvent kernel – Classical Fredholm Theory - The method of solution of Fredholm equation – Fredholm first theorem – Examples.  
**Chapter 3:** Sections 3.3 to 3.5 **Chapter 4:** Sections: 4.1 to 4.3

**Unit-IV** Applications to Ordinary Differential Equations-Initial value problems – Boundary value problems – Examples – Green's Function Approach-Examples  
**Chapter 5:** Sections 5.1 to 5.3, 5.5, 5.6

**Unit-V** Singular integral equations – The Abel integral equations – Examples, Cauchy Principal Value for Integrals-The Cauchy-Type Integrals.  
**Chapter8:** Sections 8.1 to 8.4

**Text Book:**

1. R. P. Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, NewYork,1997.

**Reference Books:**

1. Porter and Stirling, *Integral Equations*, Cambridge University press, 1996
2. M.D. Raisinghania, *Integral Equations and Boundary Value Problems*, S. Chand & Co., New Delhi, 2007.
3. Sudir K. Pundir and Rimple Pundir, *Integral Equations and Boundary Value Problems*, Pragati Prakasam, Meerut, 2005.
4. F. G. Tricomi, *Integral Equations*, Inter science Publishers, Inc Newyork, 1985

## M. Sc. MATHEMATICS

### SOFT CORE : MATH-542 ADVANCED MATHEMATICAL ANALYSIS (4 Credits )

#### Objectives:

To study the basics of Continuous functions, Uniform convergence, Hilbert spaces of holomorphic functions and Fourier transform.

#### Unit-I Spaces of functions

Families of functions like periodic functions - Continuous functions-  $C^1$ - functions- rapidly decreasing functions on  $\mathbb{R}^n$  which separate points- closed subsets - Partition of unity.

#### Unit-II Topology on the spaces functions

Uniform convergence - Uniform convergence on compact on polynomials (with emphasis on power series)-  $C^k$ -functions-  $C^1$ -functions on  $\mathbb{R}^n$  – holomorphic functions on  $\mathbb{C}$  - Completeness of various spaces of functions under uniform metric- $L_p$ -metric and under uniformly on compact topology.

#### Unit-III Compact subsets

Arzela - Ascoli theorem - Normal families of holomorphic functions - Hilbert spaces of holomorphic functions - Reproducing kernels.

#### Unit-IV Fourier analysis

Convolutions - Fourier transform - Approximate identities in  $L^1(\mathbb{R}^n)$  given by classical kernels like Fejer's kernel.

#### Unit-V Density

Approximation through convolutions - Density theorems of weierstrass and stone- Korovkin – Density of  $C^1_c$  -functions in  $L_p$ .

#### Text Books

1. R. Beals, Advanced mathematical analysis, Springer Verlag, New York, 1973.
2. J. B. Conway, Functions of one complex variable, Narosa Publishing House, 1980.
3. E. H. Lieb and M. Loss, Analysis, Narosa Book House, New Delhi, 1997.

#### Reference Books

1. W. Rudin, Real and complex analysis, 2nd ed., TMH Edition, 1962.
2. K. Yosida, Functional analysis, Springer - Verlag, New York, 1968.

## M. Sc. MATHEMATICS

### SOFT CORE : MATH-544 ELEMENTS OF HARMONIC ANALYSIS ( 4 Credits )

#### Objectives:

To study the basics of Topological groups, Connected groups, The Dual group of a locally compact abelian group and Classical kernels on  $\mathbb{R}$ .

#### Unit-I

Basic properties of topological groups, subgroups, quotient groups - Examples of Various matrix groups.

#### Unit-II

Connected groups - Existence of Haar measure (without proof) - Computation of Haar measure on  $\mathbb{R}$ ,  $\mathbb{T}$ ,  $\mathbb{Z}$  and some simple matrix groups - Convolution, the Banach algebra  $L^1(G)$  and convolution with special emphasis on  $L^1(\mathbb{R})$ ,  $L^1(\mathbb{T})$  and  $L^1(\mathbb{Z})$ .

#### Unit-III

Fourier transform and its properties - Approximate identities in  $L^1(G)$ .

#### Unit-IV

The Dual group of a locally compact abelian group - Computation of dual groups for  $\mathbb{R}$ ,  $\mathbb{T}$ ,  $\mathbb{Z}$ .

#### Unit-V

Classical kernels on  $\mathbb{R}$  - The Fourier inversion Theorem - Plancherel theorem on  $\mathbb{R}$  - Plancherel measure on  $\mathbb{R}$ ,  $\mathbb{T}$ ,  $\mathbb{Z}$  - Discussion on Plancherel theorem on a general locally compact abelian group.

#### Text Books:

1. G. Folland, A course in abstract harmonic analysis, CRC Press, 1994.
2. H. Helson, Harmonic analysis, Trim Series, Hindustan Book Agency, 2nd Edition, 1995.
3. Y. Katznelson, Introduction to harmonic analysis, J. Wiley and Sons, 1968.
4. L.H. Loomis, An introduction to abstract harmonic analysis, van Nostrand, New York, 1953.

#### Reference Books:

1. E. Hewitt & K.A. Ross, Abstract harmonic analysis, Vol. I, Springer – Verlag, 1963.
2. W. Rudin, Real and complex analysis, Tata Mc Graw Hill, 2nd Edition, 1962.

**M. Sc. MATHEMATICS**  
**SOFT CORE : MATH-545 LINEAR LIE GROUPS (4 Credits )**

**Objectives:**

To study the basics of Topological groups, Unitary groups, Computation of Haar measure of the groups and Representations of a locally compact group.

**Unit-I**

Basic properties of topological groups, subgroups, quotient groups and connected groups.

**Unit-II**

Linear Lie groups like  $GL(n, \mathbb{R})$ ,  $GL(n, \mathbb{C})$ , Orthogonal groups, Unitary groups, Motion groups, Heisenberg groups and various properties of them.

**Unit-III**

Computation of Haar measure for the above groups - The exponential map and the Lie algebras of the above groups.

**Unit-IV**

Representations of a locally compact group - Adjoint representation - Irreducible representations of  $SU(2)$  and  $SO(3)$ .

**Unit-V**

Induced representation - Irreducible representations of Motion group  $M(2)$  and Heisenberg groups.

**Text Books:**

1. J. L. Clerc, Les représentations des groupes compacts, Analyse harmonique (J.L.Clerc et al., ed.), C.I.M.P.A., 1982.
2. G. Folland, A course in abstract harmonic analysis, CRC Press, 1994.
3. S. Kumaresan, A course in differential geometry and lie groups, Trim 22, Hindustan Book Agency, 2002.

**Reference Books:**

1. M. Sugiura, Unitary representations and harmonic analysis, An introduction, John - Wiley, 1975.



**M.Sc MATHEMATICS**  
**SOFT CORE: MATH547 – ADVANCED FUNCTIONAL ANALYSIS**

**Objectives:**

To study the basics of Canonical isometry, Compact operators, Eigen values and the eigen spectrum of a linear operator, The adjoint of an operator and Spectral results for Hilbert's space operators.

**Unit-I**

Duals of  $C[a, b]$  and  $L_p[a, b]$ – Separability – The Canonical isometry – The transpose of a bounded linear Map – Reflexivity – Weak convergence – Schur's Lemma – EberleinShmulyan Theorem – Best approximation in reflexive spaces.

**Unit-II**

Compact operators – Examples – Properties – The completeness of the space of compact operators – Compactness of the transpose.

**Unit-III**

Eigen values and the eigen spectrum of a linear operator – examples – spectrum and resolvent set – Spectral radius – Spectral Mapping Theorem – Resolvent Identity – The spectral radius formula – The RieszSchauder Theory.

**Unit-IV**

The adjoint of an operator – Existence – Compactness of the adjoint operator – Sesquilinearfunctionals – Closed range Theorem.

**Unit-V**

Self-adjoint, normal, unitary operators – Numerical range and numerical radius – Spectral results for Hilbert's space operators – Properties of the Spectrum.

**Text Book:**

1. M.Thamban Nair, *Functional Analysis: A First Course*, Prentice Hall of India, 2002.

**Unit-I** Sections: 8.1.3, 8.1.4, 8.2.1, 8.2.2,8.2.3

**Unit-II** Sections: 9.1, 9.2, 9.3

**Unit-III** Sections: 10.1, 10.2, 10.2.1, 10.2.2, 10.2.3, 10.4

**Unit-IV** Sections: 11.1, 11.1.1, 11.1.2

**Unit-V** Sections: 11.2, 12.1, 12.1.1, 12.2

**Reference Books:**

1. Joseph Muscat, *Functional Analysis*, Springer (2008).
2. BalmohanV.Limaye, *Functional Analysis*,3e, New Age International Publishers (2014).
3. Erwin Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley (2007).
4. Martin Schechter, *Principles of Functional Analysis*, American Mathematical Society (2009)
5. BelaBollobas, *Linear Analysis: An Introductory Course*,2e, Cambridge Univ. Press (1999).
6. Bryan P. Rynne and Martin.AYoungson, *Linear Functional Analysis*, Springer (2008).

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-549 COMMUTATIVE ALGEBRA (4 Credits)**

**Objectives:**

To study the basics of Prime ideals, Operation on sub-modules, Tensor product and Noetherian rings.

**Unit-I**

Prime ideals- Maximal ideals- Nil radical- Jacobson radical- Operation on ideals- Extension and contraction.

**Unit-II**

Operation on sub-modules- Direct sum and product- Finitely generated modules- Exact sequences- Tensor product- Restriction and extension of Scalars.

**Unit-III**

Rings and Modules of Fraction and Primary decomposition Local properties extended and contracted Primary decomposition.

**Unit-IV**

Integral dependence and chain conditions.

**Unit -V**

Noetherian rings and Artinian rings

**Text Book**

M. K. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley, 1994.

**Reference Books**

1. H. Matsumura, Commutative Ring Theory, Cambridge University Press, 1989.
2. I. Kaplansky, Commutative Rings, University of London press, 1966.
3. O. Zariski and P. Samuel, Commutative Algebra, Springer 1976.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH 551 FUNCTIONAL ANALYSIS – II ( 4 Credits )**

**Objectives:**

To study the basics of Normed linear spaces, Weak topologies, Linear operators and Compact operators on Banach spaces

**Unit-I: Normed linear spaces**

Separation theorem and strict separation theorem in normed linear spaces-Applications-Weak and weak\* topologies on normed linear spaces, both finite and infinite dimensional

**Unit-II: Weak and weak\* topologies**

Conditions for metrizable of weak and weak\* topologies on bounded sets-Weak and weak\* continuous linear functionals-Separation theorem for spaces with weak or weak\* topologies

**Unit-III: Dual-polar set**

Weak topology induced by a subset of the dual-polar set - Bipolar theorem - Goldstein's theorem- Banach -Alaoglu's theorem- Reflexivity and weak convergence.

**Unit-IV: Operators - I**

Linear operators-Examples-Integral operators- Inverse and adjoint operators- Range and null spaces- Adjoint operators in Hilbert spaces- Normal and unitary Operators

**Unit-V: Operators - II**

Compact operators on Banach spaces- Definition, examples and basic properties- Hilbert Schmidt operators

**Text Books:**

1. M. Fabian, P.Habala, P. Hajek, V.M. Santalucia, J.Pelant and V. Zizler, Functional Analysis and Infinite Dimensional Geometry, CMS Books in Mathematics, Springer-Verlag, 2001.
2. M. Thamban Nair, Functional Analysis - A First Course, Prentice-Hall of India Private Ltd, New Delhi,2002.
3. B. V. Limaye, Functional Analysis, New- Age International Pvt. Ltd. 1996.

**Reference Books:**

1. Joseph Conway, A Course on Functional Analysis, Springer- Verlag, 1990.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH 552 OPERATOR THEORY ( 4 Credits )**

**Objectives:**

To study the basics of Banach algebras, Maximal ideal space for commutative Banach algebras, Representations of  $C^*$ - Algebras and Compact operators.

**Unit-I**

Banach algebras – Involutive Banach algebras – Various examples including Group algebras – Spectrum – Spectral mapping theorem – Spectral radius formula.

**Unit-II**

Maximal ideal space for commutative Banach algebras – Gelfand - Naimark theory for commutative Banach algebras –  $C^*$ - algebras, Examples- Commutative  $C^*$ - algebras.

**Unit-III**

Representations of  $C^*$ - Algebras – Von Neumann's density theorem – Double commutant theorem - GNS constructions.

**Unit-IV**

Functional calculus – The spectral theorem for normal operators – Spectral theorem for unitary operators – Polar decomposition.

**Unit-V**

Compact operators – Examples and properties – Spectral theorem for compact operators – Hilbert – Schmidt operators.

**Text Books:**

1. Sunder, V.S, Functional Analysis – Spectral Theory, Trim Series, Hindustan Book Agency, 1997.

**Reference Books:**

1. Takesaki, M, Theory of Operator Algebras I, Springer Verlag, 1979.
2. Yosida, K, Functional Analysis, Springer Verlag, 1968.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH - 554 NON-COMMUTATIVE RINGS AND REPRESENTATIONS**  
**(4 Credits)**

**Objectives:**

To study the basics of Modules, Semi simple rings, Structure theory of ring and substantial study of Representations.

**Unit-I Modules**

Modules - Artinian and Noetherian modules - Tensor products - Restricted and induced modules. - Indecomposable modules – Completely reducible module - Schur Lemma.

**Unit-II Radical**

Semi simple rings - The radical of a rings – The properties of Jacobson radical

**Unit-III** Group algebras - The Jacobson radical of Group Algebra – Maschke's Theorem.

**Unit-IV Structure theory**

Structure theory of ring - Density theorem - Wedderburn-Artin theorem for semi simple rings.

**Unit-V Representations**

Representations - linear representation - Matrix representation - Equivalent representation - Invariant subspaces - Irreducible representations - Direct sum of representations - Induced representation – restricted representation - Tensor product of representations - Inner products of representation.

**Text Book**

1. I. N Herstein, Non-Commutative Rings, The Mathematical Association of America, 5<sup>th</sup> Edition, 2005 (Chapter 1: Units I-III, Chapter 2: Unit IV and Chapter 5: Unit V)

**Reference Books:**

1. William Fulton and Joe Harris, Representation Theory - A First Course, Springer International Edition, Springer-Verlag, New York, 2004.
2. Jacobson, Basic Algebra II, Hindustan Publishing Corporation (India), 1983.
3. Charles W. Curtis and Irving Reiner, Representation Theory of Finite Groups and Associative Algebras, Inter Science Publishers, 1962.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-555: ADVANCED COMPLEX ANALYSIS ( 4 Credits )**

**Objectives:**

To study the basics of the space of continuous functions, Weierstrass factorization theorem and Schwarz Reflection Principle.

**Unit-I**

The space of continuous functions - Spaces of analytic functions – Spaces of meromorphic functions - The Riemann Mapping Theorem.[Chapter-7, Sections: 1,2 ,3 and 4.]

**Unit-II**

Weierstrass factorization theorem - Factorization of sine function – The gamma function - The Riemann zeta function. [Chapter-7, Sections: 5,6,7 and 8]

**Unit-III**

Runge's Theorem Simple Connectedness- Mittag Leeffler's Theorem[Chapter-8]

**Unit-IV**

Schwarz Reflection Principle - Analytic continuation along a path – Monodromy theorem. [Chapter-9 Sections 1, 2 and 3]

**Unit-V**

Jensen's formula - The genus and order of an entire function.[ Chapter-11, Sections:1,2 and 3]

**Text Book:**

1. John. B. Conway, Functions of one Complex Variable, Second Edition, Narosa Publishing House, 2002.

**Reference Books:**

1. B.C.Palka, An Introduction to the Complex function Theory, Springer, 1991.
2. H.A. Priestley, Introduction to Complex Analysis, Second Edition, Oxford University Press, 2003.
3. Donald sarason, Notes on Complex Function Theory, Hindustan Book agency ,1994.
4. L.V.Ahlfors, Complex analysis, Third Edition, McGraw Hill Book Company,1979.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-559: Mathematica Practical (4 Credits)**

**Objectives:**

To study the basics of Mathematica Software for various operations such as for Strings, Graphics, Lists and solving problems of Differential calculus and Ordinary Differential Equations.

**Unit-I**

Basic concepts: Constants- Built-In Functions- Basic Arithmetic Operations Strings- Assignment, Replacement, and Logical relations - Loops.

**Unit-II**

Two dimensional graphics and three dimensional Graphics: Plotting Functions of Single variables and Two Variables - Graphic commands.

**Unit- III**

Lists: Generating Lists- List Manipulation - Set Theory - Tables and Matrices- Equations- Algebra and Trigonometry- Polynomials.

**Unit- IV**

Differential calculus - Integral calculus - Multivariable calculus

**Unit-V**

Ordinary Differential Equations - Linear Algebra.

**Text Books :**

1. Eugene Don, Mathematica, Schaum's Outlines, Tata McGraw-Hill Edition, 2009.

**Reference Books:**

1. Bruce F. Torrence and Eve A. Torrence, Students Introductions to Mathematica, Cambridge University Press, 2008.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH: 560 - MATHEMATICAL SOFTWARE**  
**(Credits: 4)**

**Objectives:**

To study the basics of Mathematical software such as L<sup>A</sup>T<sub>E</sub>X, MATLAB and Maple.

**Unit-I**

L<sup>A</sup>T<sub>E</sub>X introduction- Installation – Math symbols and tables – TeX symbol and tables – Matrix and lists – Typing Math and text – Text environments.

**Unit-II**

Document structure – Latex Documents – The AMS articles document class – Beamer Presentation and PDF documents – Long Documents – BibteX – Make index – Books in LateX- Colours and Graphics – TeXCAD – L<sup>A</sup>T<sub>E</sub>X CAD.

**Unit-III**

Starting with MATLAB- Variables Vectors, Matrices – Creating Array in MATLAB –Menu, Workspace, working Directory, Command window, Diary, Printing- Built\_in function, User defined functions, Script M-files- Complex Arithmetic, Figen values and Eigen vectors – Two and three dimensional Plots.

**Unit-IV**

Getting around with maple – Maple input and output - Programming in Maple.

**Unit-V**

Maple: Abstract algebra – Linear algebra – Calculus on numbers – Variables- Complex Arithmetic, Eigen values and Eigen vectors – Two and three dimensional plots.

**Text Books**

1. G. Gratzner, More Math Into L<sup>A</sup>T<sub>E</sub>X, 4<sup>th</sup> edition, Springer, (2007).
2. AMOS Gilat, MATLAB an introduction with application, WILEY India Edition, (2009).
3. Brain R Hunt, Ronald L Lipsman, A Guide to MATLAB for beginners and Experienced users, Cambridge University Press. (2003)
4. Ander Heck, Introduction in Maple, Springer, (2007)



**M. Sc. MATHEMATICS**  
**SOFT CORE- MATH-562: NUMERICAL ANALYSIS (4 Credits)**

**Objectives:**

To study the basics of Convergence criterion, Consistency of system of equations and various methods to obtain numerical solutions of ordinary differential equations.

**Unit-I: Nonlinear Equations in One Variable:** Fixed point iterative method – convergence Criterion -Aitken's  $\Delta^2$ - process - Sturm sequence method to identify the number of real roots – Newton - Raphson's methods convergence criterion Ramanujan's Method - Bairstow's Method.

**Unit-II: Linear and Nonlinear System of Equations:** Gauss eliminations with pivotal strategy Jacobi and Gauss Seidel Itervative Methods with convergence criterion. LU - decomposition methods – (Crout's, Choleky and DeLittle methods) – consistency and ill conditioned system of equations - Tri-diagonal system of equations – Thomas algorithm. Iterative methods for Nonlinear system of equations, Newton raphson, Quasi newton and Over relaxation methods for Nonlinear system of equations.

**Unit-III: Interpolation:** Lagrange- Hermite- Cubic-spline's (Natural, Not a Knot and Clamped)- with uniqueness and error term, for polynomial interpolation- Bivariate interpolation- Orthogonal polynomials Grams Schmidth Orthogoralization procedure and least square- Chebyshev and Rational function approximation.

**Unit -IV: Numerical Integration:**

Gaussian quadrature, Gauss-Legendre- Gauss-Chebeshev formulas- Gauss Leguree, Gauss Hermite and Spline intergation – Integration over rectangular and general quadrilateral areas and multiple integration with variable limits.

**Unit-V: Numerical solution of ordinary differential equations:**

Initial value problems- Picard's and Taylor series methods – Euler's Method- Higher order Taylor methods - Modified Euler's method - Runge Kutta methods of second and fourth order – Multistep method - The Adams - Moulton method - stability - (Convergence and Truncation error for the above methods). Boundary - Value problems – Second order finite difference and cubic spline methods.

**Text books**

1. M. K. Jain, S. R. K. Iyengar and R.K. Jain, Numerical methods for scientific and Engineering computation, Wiley Eastern Ltd. 1993, Third Edition.
2. C. F. Gerald and P.O. Wheatley, Applied Numerical Methods, Low- priced edition, Pearson Education Asia 2002, Sixth Edition.
3. M. K. Jain, Numerical solution of differential equations, Wiley Eastern (1979), Second Edition.

**Reference books**

1. S. C. Chapra and P.C. Raymond, Numerical Methods for Engineers, Tata McGraw Hill, New Delhi, 2000
2. S. S. Sastry , Introductory methods of Numerical analysis, Prentice - Hall of India, New Delhi, 1998.
2. Kendall E. Atkinson, An Introduction to Numerical Analysis(2<sup>nd</sup> Edition), Wiley, 2008.

**M.Sc. MATHEMATICS**  
**SOFT CORE: MATH-563: INTEGRAL TRANSFORMS**  
**(4 CREDITS)**

**Objectives:**

To study the basics of Laplace transforms, Inversion Integral and Properties of Fourier transforms.

**Unit- I**

Laplace transforms -Important properties- Simple Applications- Asymptotic Properties- Watson's Lemma.

**Unit- II**

Inversion Integral- The Riemann- Lebesgue Lemma- Dirichlet Integrals- the Inversion- Watson's Lemma for loop integrals- Heaviside series expansion.

**Unit- III**

Application to ordinary differential equations- Elementary examples- Higher order equations-Partial differential equations- Heat diffusion integral equations.

**Unit-IV**

Fourier transforms- Exponential- Sine and Cosine transforms- Important properties- Spectral analysis.

**Unit- V**

Partial differential equations- Potential problems-Water waves-Basic equations-Waves generated by a Surface displacement.

**Text Book**

1. B. Davies, Integral Transforms and Their Applications, Springer, Texts in Applied Mathematics, 41 Third Edition, 2009.

**Reference Books:**

1. Ian N. Snedden, The Use of Integral Transforms, McGraw Hill, 1972.

**M.Sc MATHEMATICS**  
**SOFT CORE: MATH: 565 - DYNAMICAL SYSTEMS ( 4 Credits)**

**Objectives:**

To study the basics of Planar systems and its classifications, Nonlinear systems, Stability and Hamiltonian systems and Bifurcations.

**Unit-I** ( Chapters 1, 2 and 3 of [1] )

First order differential equations, The logistic population model, Second order differential equations, planar systems, Planar linear systems, Solving linear systems, The linearity principle, Phase portraits for planar systems.

**Unit-II** ( Chapters: 4 and 6 of [1] )

Classification of planar systems, higher dimensional linear systems- Harmonic oscillators -The exponential of a matrix- Nonautonomous linear system.

**Unit-III** ( Chapters 7 and 8 of [1] )

Nonlinear systems-Dynamical systems-The existence and uniqueness theorem-continuous dependence of solutions- The variational equation. Equilibria in nonlinear systems- Nonlinear sinks and sources-saddles and stability-bifurcations.

**Unit-IV** (Chapters 9 and 10 of [1] )

Global nonlinear Techniques- Nullclines -Stability of equilibria – Gradient systems- Hamiltonian systems, closed orbits and limit sets-Local sections and flow boxes- The Poincare Map- Monotone sequences in planar Dynamical systems- The Poincare\_ Bendixson theorem- Applications.

**Unit-V** (Chapter 15 of [1] )

Discrete dynamical systems-Bifurcations-The discrete logistic model- Chaos- Symbolic dynamics-The shift map –The Cantor middle\_third set.

**Text Book:**

1. Morris W.Hirsch, Stephen Smale, Robert L.Devaney, Differential Equations and Dynamical systems and An Introduction to Chaos, Second edition, Academic Press(Elsevier) 2004.

**Reference Books:**

1. Robert L.Devaney, A First Course in Chaotic Dynamical Systems, Addison-Wesley Publishing Company, Inc. 1992.
2. Lawrence Perko, Differential equations and Dynamical Systems, (3<sup>rd</sup> Edition), Springer 2001.

## M.Sc MATHEMATICS

### SOFT CORE : MATH: 566 - ADVANCED TOPOLOGY (4 Credits)

#### Objectives:

To study the basics of Connected components, Topological groups, Compactification, Dimension theory and Homotopy of paths.

#### Unit-I (Sections- 25, 29 and 34 of [1])

Connected components- Local connectedness - Locally path connected spaces- Local compactness, One point Compactification, Uryshon Metrization Theorem.

#### Unit-II (Chapter-10 of [2] and Sections- 22 and 36 of [1] )

Nets and Filters- Quotient topology- Introduction to topological groups- Existence of partition of unity- Imbedding theorem for compact m- manifolds.

#### Unit-III (Sections-38,39,40,41 and 42 of [1] )

The Stone -Cech Compactification- Locally finite spaces- Nagata- Smirnov Metrization theorem- Para compactness- Smirnov Metrization theorem.

#### Unit-IV (Sections-44 and 50 of [1] )

The Peano space-filling curve- Introduction to dimension theory- Imbedding theorem for compact Metrizable spaces.

#### Unit-V (Sections- 51, 52, 53 and 54 of [1] )

Homotopy of paths- The fundamental group- Covering spaces- The fundamental group of the circle.

#### Text Books:

1. James R. Munkres, Topology, Second edition, Pearson Education Inc.,(2002).
2. K. D. Joshi, Introduction to General Topology, First edition (revised), New Age International Publishers, 2004.

#### Reference Books:

1. Stephen Willard, General Topology, Dover, 2004.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH: 567-SPECIAL FUNCTIONS IN NUMBER THEORY**  
**(4 Credits)**

**Objectives:**

To study the basics of Hyper Geometric series, Jacobi's triple product identity and its applications, q- series and theta functions and Modular relation related to hypergeometric series.

**Unit I**

Introduction to Basic hyper Geometric series- Binomial theorem- q- binomial theorem Heine's Transformation formula- Jackson transformation formula

**Unit II**

Jacobi's triple product identity and its applications and Quintuple product identity and new identity for  $(q; q)_{\infty}^{10}$  with application to partition congruence modulo 11

**Unit III**

Bilateral Series- Ramanujan I  $\psi$  1 summation and related identities- Ramanujan theta function identities involving Lambert series.

**Unit IV**

q- series and theta functions Entries 18 to 30 Chapter 16 of Ramanujan's notebook.

**Unit V**

Modular relation related to hypergeometric series and its applications.

**Text Book**

1. Gasper and Rahman, Basic hyper geometric series, Cambridge University press 1990.(Unit I-III)
2. BC Berndt Ramanujan's notebooks Part II Springer Verlag New York 1991.(Unit IV-V)

## M.Sc. MATHEMATICS

### SOFT CORE: MATH: 568 -THEORY OF PARTITIONS (4 Credits)

#### Objectives:

To study the basics of Hyper geometric series, Jacobi's triple product identity, Partition functions and Rank and crank of partitions.

#### Unit I:

Introduction to basic hyper geometric series, q- binomial theorem, Heine's transformation and Gaussian Polynomial.

#### Unit II:

Jacobi's triple product identity and its applications, bilateral series and its applications, theta functions.

#### Unit III:

Partition function and its generating function, Euler theorem for partition.

#### Unit IV:

Congruence properties of partition functions, Rogers Ramanujan Identities.

#### Unit V:

Rank and crank of partitions and restricted partitions.

#### Text Books

1. Bruce C. Berndt, Number Theory in the Spirit of Ramanujan, AMS (For unit I and II)
2. G. E. Andrews, The Theory of Partitions, Addison Wesley 1979.(For unit III, IV & V)

#### Reference Books:

1. Gasper and Rahman, Basic hypergeometric Series, Cambridge University Press 1990.
2. G.E Andrews, R. Askey and Ranjan Roy, Special functions, Cambridge University press 2000.
3. Bruce C. Berndt, Ramanujan's Notebooks Vol III, Springer, New York 1991.

**M. Sc. MATHEMATICS**  
**SOFT CORE: MATH-570: Introduction to Fuzzy Set (4 credits)**

**Objectives:**

To study the basics of Fuzzy sets, Operations on Fuzzy sets, Functions on Fuzzy sets, Fuzzy controller and Fuzzy Data Analysis.

**Unit 1: Basics on Fuzzy Set**

Crispness-Vagueness-Fuzziness-Uncertainty-Fuzzy Set Theory- -Theoretic Operations for Fuzzy Set- Types of Fuzzy Sets- Operations on Fuzzy Set- Algebraic Operations- Set-Theoretic Operations.

**Unit 2: Generalization of crisp mathematical concepts to fuzzy sets**

Operations for Type2 Fuzzy Sets- Algebraic Operations with Fuzzy Numbers- Special Extended Operations- Extended Addition- Extended Product- Extended Subtraction- Extended Division- Extended Operations for LR-Representation of Fuzzy Sets.

**Unit 3: Fuzzy Analysis**

Fuzzy Functions on Fuzzy Sets-Integration of Fuzzy Functions-Integration of a Fuzzy Function over a Crisp Interval-Some Properties of Integrals of Fuzzy Functions-Integration of a (Crisp) Real-Valued Function over Fuzzy Interval-Fuzzy Differentiation.

**Unit 4: Fuzzy Control**

Origin and Objective-The Fuzzy Controller-Types of Fuzzy Controllers-The Mamdani Controller-Defuzzification-The Sugeno Controller- Design Parameters-Scaling Factors-Fuzzy Sets-Rules-Adaptive Fuzzy Control.

**Unit 5: Fuzzy Data Analysis**

Methods for Fuzzy Data Analysis-Algorithmic Approaches- Knowledge-Based Approaches-Dynamic Fuzzy Data Analysis- Similarity of Functions.

**Text Book:**

H.J. Zimmermann: Fuzzy set theory and its Applications, Springer Science + Business Media New York, 2001.

**Reference Books:**

George J. Klir and Bo Yuan: Fuzzy sets and Fuzzy Logic: Theory and Applications, Prentice – Hall of India Private Limited, New Delhi, 2000.

George Bojadziev and Maria Bojadziev, Fuzzy Sets, Fuzzy Logic, Applications, World Scientific Publishing Co.Pte.Ltd, Singapore, 1995.

## M.Sc. MATHEMATICS

### SOFT CORE: MATH-571 -CALCULUS OF VARIATIONS ( 4 Credits)

#### Objectives:

To study the basics of Functionals, Functionals depending on higher order derivatives, Variational problems, Noether's Theorem and Conservation laws.

**Unit I:** Functionals- some simple variational problems-The variation of a functional- A necessary condition for an extremum- The simplest variational problem-Euler's equation-The case of several variables-A simple variable end point problem- The variational derivative-Invariance of Euler's equation. [Chapter-1]

**Unit II:** The fixed end point problem for  $n$ -unknown functions - Variational problem in parametric form- Functionals depending on higher order derivatives-Variational problems with subsidiary conditions. [Chapter-2]

**Unit III:** The general variational of a functional- derivation of the basic formula- End points lying on two given curves or surfaces- Broken extremals- The Weierstrass Erdmann conditions. [Chapter-3]

**Unit IV:** The canonical form of Euler equations- First integrals of the Euler equations- The Legendre transformation- Canonical transformations- Noether's Theorem- The principle of least action- Conservation laws- The Hamilton Jacobi equation- Jacobi theorem. [Chapter-4]

**Unit V:** The second variation of a functional- The formula for the second variation, Legendre conditions- Sufficient conditions for a weak extremum.. [ Chapter-5]

#### Text Book:

I.M. Gelfand and S.V.Fomin, *Calculus of Variations*, Dover Publications, 2000.

#### Reference Books:

1. A.S. Gupta, *Calculus of Variations with Applications*, Prentice-Hall of India, 2008.
2. M.L. Krasnov, G.I. Makarenko and A.I. Kiselev, *Problems and Exercises in the Calculus of Variations*, Mir Publishers, Moscow 1975.



**M. Sc. MATHEMATICS**  
**SOFT CORE:- MATH-572: Probability and Statistics (4 Credits)**

**Objectives:**

To study the basics of Probability density function, Special distributions, Distributions of functions of random variables, Sampling theory and Statistical inference.

**Unit I:** The probability set function – Random variables – Probability density function – Distribution function – Mathematical expectation – Special mathematical expectations – Chebyshev's Inequality – Conditional probability – Marginal and conditional distributions – Stochastic independence. [ Chapters 1 and 2 (except 1.1 and 1.2 ) of the text book ]

**Unit II:** Some special distributions: The Binomial and related distributions – The Poisson distribution – The Gamma and Chi-Square Distributions – The Normal distribution- The Bivariate normal distribution. [Chapter -3 of the text book]

**Unit III:** Distributions of functions of random variables - Sampling theory – Transformations of variables of the discrete type – Transformations of variables of the continuous type – The  $b, t$  and  $F$  distributions- Distributions of order statistics- The moment generating function technique. [Chapter 4 [ sections 4.1 to 4.7 ] of the text book.]

**Unit IV:** The distributions of  $\bar{X}$  and  $nS^2/\sigma^2$  - Expectations of functions of random variables – Limiting distributions: Limiting moment generating functions – The Central limit theorem. [Chapter-4 [sections 4.8 and 4.9] and Chapter-5 of the text book.]

**Unit V:** Introduction to statistical inference: Point Estimation – Confidence intervals for means – Confidence intervals for differences of means - Confidence intervals for variances. [Chapter-6 of the text book]

**Text Book:**

Robert V. Hogg and Allen T. Craig , *Introduction to Mathematical Statistics* (Fifth Edition) Pearson Education, 2005.

**Reference Books:**

1. Paul L.Meyar, *Introductory to Probability and Statistical Applications*, Oxford&IBH Publishers Co. Pvt .Ltd, 1969.
2. Arnold Naiman, Gene Zirkel and Robert Rosenfield, *Understanding Statistics*, McGraw-Hill, 1986.
3. William Feller, *An Introduction to Probability Theory and its Applications, Vol.I*, John Wiley, Third Edition, 2008.
4. A.Mood, F.Graybill, and D.Boes, *Introduction to the Theory of Statistics*, Tata McGraw Hill (Third Edition) 2008.